RADIANT HEAT TRANSFER AND HEAT CONDUCTION IN PARALLEL PLATES WITH LATERAL SUPPLY (REMOVAL) OF HEAT

A. G. Golubchin

UDC 536.21:536.33:518.3

An analysis is given of radiative and conductive heat transfer in parallel plates with lateral supply (removal) of heat. An approximation method for the solution of heat-transfer equations is presented, and the results of the calculations are represented by nomograms.

In the process of thermal treatment of thin ceramic products collected into a cassette or hollow ceramic tiles, the transfer of heat takes place by heat conduction in the mass of the product and by radiation across the gap.

For a mathematical description of the conditions of heat exchange, we construct balanced equations of heat transfer by thermal conduction and radiation with the following assumptions: the supply (removal) of heat is symmetrical, from the ends; the temperature of the heater (cooler) is equal to the temperature of the end of the product T_1; the emissivity of the surfaces taking part in the heat exchange is equal to unity; the convective component of the heat exchange is small and is not considered in the calculation.

We examine the conditions of heat exchange in the system (Fig. 1). For the analysis we select elementary areas dx and dy on the lower and upper surfaces of the gap. Heat arrives at the area dx by radiation from the heaters I and II and the upper surface of the gap, as well as by heat conduction through the plate. Heat is dispersed in radiation of the area and by the internal heat absorption q_x. The heat balance between the elementary area dx and the surrounding medium is written in the following form:

$$\sigma_0 T_x^4 dx + q_x dx = \sigma_0 T_1^4 h (\varphi_{I,dx} + \varphi_{II,dx}) + \sigma_0 \int_0^l T_y^4 dy \varphi_{dy,dx} + \lambda \delta_1 \frac{d^2 T_x}{dx^2} dx. \qquad (1)$$

In accordance with the principle of reversibility of angular coefficients,

$$h\varphi_{I,dx} = dx\varphi_{dx,I}; \quad h\varphi_{II,dx} = dx\varphi_{dx,II}; \quad dy\varphi_{dy,dx} = dx\varphi_{dx,dy}.$$

We determine the angular coefficients by the method proposed by Jacob [1, 2]:

$$\varphi_{dx,I} = \frac{1}{2}\left(1 - \frac{l-x}{\sqrt{h^2 + (l-x)^2}}\right);$$

$$\varphi_{dx,II} = \frac{1}{2}\left(1 - \frac{x}{\sqrt{h^2 + x^2}}\right);$$

$$\varphi_{dx,dy} = \frac{1}{2} \cdot \frac{h^2}{[h^2 + (x-y)^2]^{3/2}} dy.$$

Substituting the expressions for the angular coefficients $\varphi_{dx,I}$, $\varphi_{dx,II}$, and $\varphi_{dx,dy}$ into Eq. (1) we obtain the first equation of the system

State Scientific-Research Institute of Building Materials and Products, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 2, pp. 213-218, August, 1971. Original article submitted November 13, 1970.

$$\sigma_0 T_x^4 + q_x = \sigma_0 T_1^4 \left(1 - \frac{l-x}{2\sqrt{h^2+(l-x)^2}} - \frac{x}{2\sqrt{h^2+x^2}}\right) + \frac{\sigma_0}{2}\int_0^l T_y^4 \frac{h^2}{[h^2+(x-y)^2]^{3/2}}\,dy + \lambda\delta_1 \frac{d^2T_x}{dx^2}. \tag{2}$$

We reduce the equation to a suitable dimensionless form with the help of the variables

$$\theta = \frac{T}{T_1};\quad X = \frac{x}{l};\quad Y = \frac{y}{l};\quad \bar{q}_X = \frac{q_x}{\sigma_0 T_1^4};$$

$$\theta_X^4 + \bar{q}_X = 1 - \frac{1-X}{2\sqrt{\left(\frac{h}{l}\right)^2 + (1-X)^2}} - \frac{X}{2\sqrt{\left(\frac{h}{l}\right)^2 + X^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\theta_Y^4}{\left[\left(\frac{h}{l}\right)^2 + (X-Y)^2\right]^{3/2}}\,dY + \frac{1}{N_c'}\cdot\frac{d^2\theta_X}{dX^2},$$

where $N_c' = \sigma_0 T_1^3 l^2/\lambda\delta_1$ is a thermophysical parameter representing the ratio of the radiation intensity of the surface of the gap at the temperature T_1 to the normal component of the heat conduction of the plate; h/l is a geometrical parameter (the ratio of the height of the gap to its width).

We obtain analogous equations for the composition of the heat balance between the elementary area dy and the surrounding medium

$$\theta_Y^4 + \bar{q}_Y = 1 - \frac{1-Y}{2\sqrt{\left(\frac{h}{l}\right)^2 + (1-Y)^2}} - \frac{Y}{2\sqrt{\left(\frac{h}{l}\right)^2 + Y^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\theta_X^4}{\left[\left(\frac{h}{l}\right)^2 + (Y-X)^2\right]^{3/2}}\,dY + \frac{1}{N_c''}\cdot\frac{d^2\theta_Y}{dY^2};\quad N_c'' = \frac{\sigma_0 T_1^3 l^2}{\lambda\delta_2}. \tag{3}$$

If plates of identical thickness $\delta_1 = \delta_2 = \delta$ are examined, then $\theta_X = \theta_Y$, $\bar{q}_X = \bar{q}_Y$, $N_c' = N_c'' = N_c$. In this case in place of the system of equations the heat-exchange conditions may be written in one nonlinear integrodifferential equation

$$\theta_X^4 + \bar{q}_X = 1 - \frac{1-X}{2\sqrt{\left(\frac{h}{l}\right)^2 + (1-X)^2}} - \frac{X}{2\sqrt{\left(\frac{h}{l}\right)^2 + X^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\theta_X^4}{\left[\left(\frac{h}{l}\right)^2 + (X-Y)^2\right]^{3/2}}\,dX + \frac{1}{N_c}\cdot\frac{d^2\theta_X}{dX^2}. \tag{4}$$

There are two unknowns in this equation; $\bar{q}_X$ and θ_X.

To solve the equation it is necessary, first, to represent the heat flux $\bar{q}_X$ in the form of a function of other variables or to assign it a constant value, and second, in place of the relative temperature θ_X in the integrand to substitute an approximation formula expressing the distribution of relative temperatures along the edge of the gap.

As a result of the studies conducted on the experimental apparatus it was established that the distribution of relative heat fluxes along the edge of the gap in the principal heating and cooling zones is expressed by the formula $\bar{q}_X = \bar{q}_c = \text{const}$. For the conditions of exposure at the maximum firing temperature $\bar{q}_X = \bar{q}_c \sin \pi X$. The nature of the distribution of relative temperatures along the edge of the gap during the heating period has the following dependence: $\theta_X = 1 - \Delta\theta_X \sin \pi X$, where $\Delta\theta_X$ is the relative temperature drop allowed because of the generation of thermal stresses in the product [3].

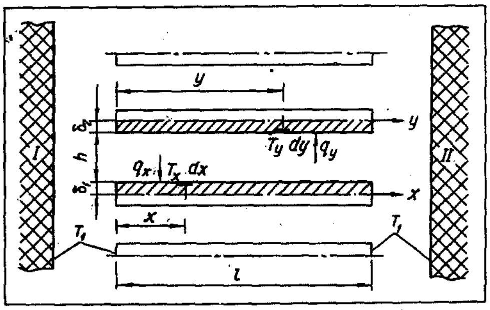

Fig. 1. Heat-exchange diagram. I, II) Heaters (coolers).

After substituting the expressions obtained for $\bar{q}_X$ and θ_X in Eq. (4), and considering that the given equation is accurate for all values of X and Y from 0 to 1 and consequently for X and Y equal to 0.5, we obtain a nonlinear algebraic equation of the following form:

$$(1-\Delta\theta_X)^4+\bar{q}_c=1-\frac{0.5}{\sqrt{\left(\frac{h}{l}\right)^2+0.5^2}}+\frac{\left(\frac{h}{l}\right)^2}{2}\int\limits_0^1\frac{(1-\Delta\theta_X\sin\pi X)^4}{\left[\left(\frac{h}{l}\right)^2+(X-0.5)^2\right]^{3/2}}dX+\frac{\pi^2\Delta\theta_X}{N_c}, \quad (5)$$

the solution of which is presented in nomogram 1 (Fig. 2) for a wide range of variation of the parameters $\Delta\theta_X$, h/l, and N_c.

This nomogram is accurate both for the condition $\bar{q}_X \cong \bar{q}_c$ = const, which allows the calculation of permissible heating temperatures, and for the condition $\bar{q}_X \cong \bar{q}_c \sin\pi X$, as a result of which one can determine the time necessary for equalization of the temperature along the edge of the plate during the exposure period.

The nomogram is composite: the value of the heat flux contributing to the heat-conduction portion is determined in the right-hand part, and the radiant component of the heat flux on the left.

We determine on the nomogram the temperature T_1 at which the amounts of heat supplied to the elementary area by heat conduction and by radiation are equal.

The initial data are: $\Delta\theta_X = 0.1$; $h/l = 0.1$; $l = 0.15$ m; $2\delta = 0.010$ m; $\lambda = 1.0$ W/m² · deg.

In the right-hand part of the nomogram we determine the value of $\bar{q}_r$, which is equal to 0.025. In the left-hand part of the nomogram for $\Delta\theta_X = 0.1$ and $\bar{q}_r = \bar{q}_t = 0.025$ the thermophysical parameter N_c is equal to 40.

The temperature of the end of the plate is

$$T_1=\sqrt[3]{\frac{N_c\lambda\delta}{\sigma_0 l^2}}=\sqrt[3]{\frac{40\cdot 1\cdot 0.005}{5.67\cdot 10^{-8}\cdot 0.15^2}}=540\,°\text{K}.$$

At a temperature $T_1 = 1270$°K, using a value of λ of about 1.4 W/m² · deg, the thermophysical parameter $N_c = 480$, which according to nomogram 1 at the initial parameters $\Delta\theta_X = 0.1$ and $h/l = 0.1$ corresponds to $\bar{q}_t = 0.002$, i.e., at the given heater temperature the amount of heat entering the elementary area of the plate surface from radiation is higher by a factor of 12 than the heat introduced by heat conduction.

In an analysis of the conditions of cooling, using the discussion presented above, we obtain a nonlinear integrodifferential equation describing in general form the conditions of cooling:

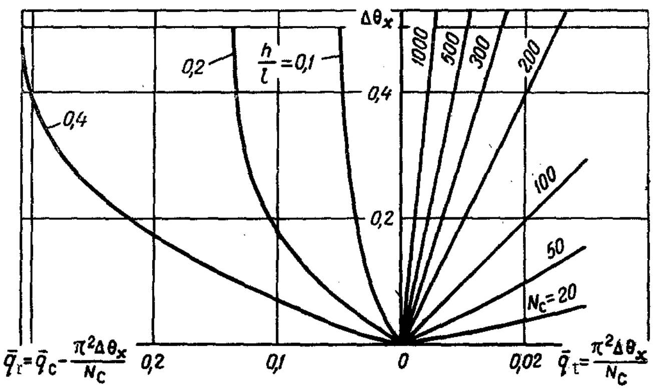

Fig. 2. Nomogram 1 for calculation of heat consumption during the heating period. $\bar{q}_c = \bar{q}_r + \bar{q}_t$; $\bar{q}_r = \bar{q}_c - \pi^2\Delta\theta_X/N_c$; $\bar{q}_t = \pi^2\Delta\theta_X/N_c$.

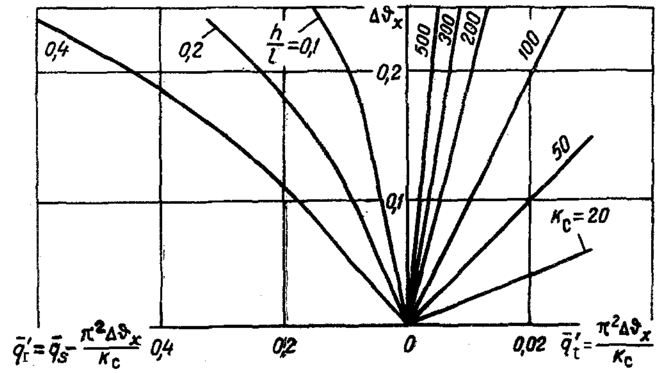

Fig. 3. Nomogram 2 for calculation of heat loss under conditions of cooling. $\bar{q}_s = \bar{q}_r' + \bar{q}_t'$; $\bar{q}_r' = \bar{q}_s - \pi^2\Delta\vartheta_X/K_c$; $\bar{q}_t' = \pi^2\Delta\vartheta_X/K_c$.

$$\vartheta_X^4 + \frac{d^2\vartheta_X}{K_c dX^2} = 1 - \frac{1-X}{2\sqrt{\left(\frac{h}{l}\right)^2+(1-X)^2}} - \frac{X}{2\sqrt{\left(\frac{h}{l}\right)^2+X^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\vartheta_X^4}{\left[\left(\frac{h}{l}\right)^2+(X-Y)^2\right]^{3/2}}dX + \bar{q}_{X'}, \tag{6}$$

where $K_c = \sigma_0 T_l^3 l^2/\lambda\delta$.

Substituting into Eq. (6) functions expressing the nature of the distribution of relative temperatures and heat fluxes along the edge of the gap under conditions of cooling: $\vartheta_X = 1 + \Delta\vartheta_X \sin\pi X$ and $\bar{q}_{X'} = \bar{q}_s = \text{const}$, a nonlinear algebraic equation results:

$$(1+\Delta\vartheta_X)^4 + \frac{\pi^2\Delta\vartheta_X}{K_c} = 1 - \frac{0,5}{\sqrt{\left(\frac{h}{l}\right)^2+0.5^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{(1+\Delta\vartheta_X\sin\pi X)^4}{\left[\left(\frac{h}{l}\right)^2+(X-0.5)^2\right]^{3/2}}dX + \bar{q}_s. \tag{7}$$

The solution of Eq. (7) within the limits of variation of the parameters $\vartheta_X = 0$–0.25, $h/l = 0.1$–0.4, and $K_c = 20$–500 is presented in Fig. 3.

NOTATION

T	is the absolute temperature;
θ, ϑ	are relative temperatures;
$\Delta\theta$	is the drop in relative temperature across the plate thickness during the heating period;
$\Delta\vartheta$	is the drop in relative temperature across the plate thickness during cooling;
q	is the rate of heat consumption or heat exchange;
$\bar{q}$	is the relative rate of heat consumption or heat exchange;
h	is the separation between plates;
δ	is the half-width of the plate;
l	is the plate thickness;
λ	is the coefficient of heat conduction;
φ	is an angular coefficient;
σ_0	is the radiation constant of an absolutely black body;
x	is the coordinate along the upper surface of the plate;
y	is the coordinate along the lower surface of the plate;
X, Y	are dimensionless coordinates.

Subscripts

I, II are radiator (cooler) numbers;
x denotes the upper surface of the plate;
y denotes the lower surface of the plate;
c denotes the constant total value of the heat flux during heating period;
s denotes the constant total value of the heat flux during cooling;
r denotes the radiant component of heat flux;
t denotes the conductive component of the heat flux.

LITERATURE CITED

1. M. Jacob, Problems of Heat Transfer [Russian translation], IL (1960).
2. E. M. Sparrow, Heat Transfer [Russian translation], Vol. 85, Ser. C, No. 2, IL (1963).
3. A. E. Erinov and A. G. Golubchin, Inzh.-Fiz. Zh., 16, No. 5 (1969).